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Inquiry as an Entry Point to Equity in the Classroom

Although many policy documents include equity as part of mathematics education standards and principles, researchers continue to explore means by which equity might be supported in classrooms and at the institutional level. Teaching practices that include active learning have been proposed to address equity. In this paper, through aligning some characteristics of inquiry put forth by Cook, Murphy, and Fukawa-Connelly [1] with Gutiérrez's [2] dimensions of equity, we theoretically explore the ways in which active learning teaching practices that focus on inquiry could support equity in the classroom.

Keywords: active learning, equity, Inquiry-based Learning, Inquiry-oriented Instruction

Many curriculum and policy documents, as well as research studies, highlight the importance of equity and caution educators of the possible consequences of not attending to those issues in teaching. Most recently, the topic study group on *Equity in Mathematics Education* at the International Congress of Mathematics Education [3] raised the question: 'In the context of diversity of student populations in many classrooms around the world, how do we understand and promote equity that goes beyond mere academic and critical deliberations towards policy and practice?' [p.3]. Similarly, Gutiérrez [4] indicated, 'Most members of the mathematics education research community would agree that equity is a valued goal... However, much less consensus arises when the question is raised: how do you think we should address equity?' [p.2]

Addressing equity in mathematics education is a multi-dimensional problem (considering classroom, institutional, and systemic issues) that may require multiple approaches. In this paper, we focus on teaching practices – a dimension that can be influenced by instructors in the classroom. We specifically explore 'inquiry' teaching practices that could potentially address

issues regarding equity [5,6]. Teaching using inquiry (e.g., Inquiry-Based Learning (IBL) or Inquiry-Oriented Instruction (IOI)) has been shown to have positive effects on:

- conceptual understandings of central ideas [7,8],
- affective traits such as all students' (but especially women's) confidence in doing and teaching mathematics, interests in pursuing mathematics, attitudes about mathematics, persistence [8], and self-, cognitive, and social empowerment ([9]).

In addition, there are results that indicate active learning can benefit a greater range of students without negatively impacting traditionally high-achieving students [8], which addresses the 'excellence vs. equity debate' [10,p.148-149]. Such results, as well as our own classroom teaching experiences, encouraged us to explore the connections between equity and inquiry teaching practices theoretically. In this paper, we propose a theoretical framework to support and explore the effects of inquiry in relation to equity. With this framework, we claim that many of the characteristics of inquiry teaching put forth by Cook, Murphy, and Fukawa-Connelly [1] align with the Four Dimensions of Equity proposed by Gutiérrez [2]. That is, we claim the four dimensions – access, achievement, identity, and power– explicate how inquiry pedagogy could promote equity in mathematics courses.

Motivation

As part of a larger study concerning fostering mathematical creativity in the classroom, our research team conducted interviews in an undergraduate introduction-to-proofs course taught using IBL. The course was taught at a private Hispanic-Serving Institution in the United States where the student population is predominately female and/or first generation¹.

¹ Though the definition varies, we use the meaning that no earlier generations have received a college degree from any institution in the world.

During analysis of the interview data for one of the creativity research projects, the researchers noticed responses that were related to issues of equity in the classroom. Students mentioned voice (as a metaphor for expressing opinions or thoughts), questioning authority, and confidence in mathematics. For example (emphasis added by authors),

Vana²: I feel like all of us, you know there was some strong students in the class that kept coming up, but then I saw the quieter ones also **get their voice** during the semester (Latina, adult learner, first-generation, university staff, biology (degree completed))

Ahn Pan: [B]ecause of the nature of how the course was conducted, **it encourages questions**... you know **question authority and don't take anything for granted and, you know fight back**. (Male, White and Asian, adult learner, chemistry (math minor))

Peyton: [B]ecause of the nature of this course, ... when I did finally understand something, I did feel like I had a way stronger, **I had much more confidence in it than I do generally** and I retained the information a lot more ... I barely even reviewed anything and I still remembered it by the end of the year. (Female, White, traditional-aged transfer, first-generation, economics major)

While the above quotes referenced the nature of the course, students also detailed specific characteristics of the instructor's actions as they experienced them.

² These are either self-chosen pseudonyms or chosen by researchers when there was no indicated pseudonym.

- *Students presented and evaluated each other's work.* Cargo: I think just having my classmates just **go up and share their work** and their thought process helped me see things, I didn't notice. Even when **I was up presenting**, there was always one guy that would always just keep asking '**How did you get that?**' And, because he kept asking that, I kept figuring out 'OK. I think I should probably put more details into my proofs so they know where I'm getting these things'. (Latino, traditional-aged, first generation, math major)
- *Students engaged in group and whole class discussions on tasks assigned by instructor.* Alice: She would **assign homework** and then we'd always discuss them in class...being able to have those **class discussions** as well as like our **individual group discussions** that we had in class. (Latina, traditional-aged, first generation, math major)
- *Instructor had a modified role from the traditional lecturer.* Vana: The instructor was very, I don't know if limited is the right word in terms of her involvement in class...[she] kind of sat at the table and more was a **listener and a mediator**, like a **facilitator of our discussions** but she never really led the discussion. So it was a lot of you know **bouncing ideas off of students** and kind of **evaluating each other's work** in that sense. (Latina, adult learner, first-generation, university staff, biology (degree completed))

These student responses motivated the research group to consider the connection between learning through inquiry and equity. To explore this possible connection, we examined definitions and frameworks for equity teaching in the mathematics education literature, which we share a summary of in the next section.

Equity

In general, equity teaching promotes a mindset where all students are capable of learning mathematics [10-12]. Equity research seeks to surface teaching practices that enable these mindsets [10] among instructors and students alike [13]. It is important that instructors bracket prejudices about student participation and achievement levels based on race, gender, social class, proficiency in the dominant language, ethnicity or other characteristics [10]. Similarly, judgments based on a student's prior performance, particularly if they have performed poorly in the past, should not be seen as a personal weakness. Rather, we as instructors need to recognise that their level of performance could be a consequence of the complex social, economic, and cultural factors [14] that affect individual experiences while learning mathematics.

For the purposes of this theoretical investigation, we utilise an equity framework used in previous studies [15]. Gutiérrez [2] argued that teaching for equity includes four dimensions: *Access, Achievement, Identity* and *Power*. *Access* and *Identity* are considered precursors to *Achievement* and *Power*, respectively. On one axis, *Access* addresses the resources that have been made available for students to participate in mathematics such as 'quality of teachers, adequate technology and supplies, classroom environment that invites participation, infrastructure for learning outside the classroom' [p.5], and the opportunities to draw upon their 'cultural and linguistic resources' [p.5]. On the other end of the same axis, *Achievement* is an outcome affected by students' access to opportunities to learn and can be measured by 'participation in class, course-taking patterns, standardised test scores, majoring in math, [or] having a math-based career' [p.5]. Adiredja, Alexander and Andrews-Larson [15] summarised this description by offering that learning outcomes can range from the 'knowledge on specific content to students' ability to productively use mathematics to participate in society' [p.64].

On a different axis, *Identity* attends to the ‘balance between self and the global society and ways students are racialized, gendered and classed’ [2,p.5], where attention needs to be paid ‘to whose perspectives and practices are “socially valorized”’ [p.5]. The goal is to ‘strike a balance between opportunities to reflect on oneself and others as part of the mathematics learning experience’ [p.5]. Gutiérrez explained *Power* as students using their math knowledge to reach ‘personal goals of excellence such as helping their community to solve a local problem’ [p.6]. Adiredja et al. [15] added that learning focused on this dimension attends to ‘disrupting the existing power distribution and dynamics in a society based on race, gender, and social class’ [p.64]. To achieve this, students can be involved in decision-making on ‘what counts as productive mathematical knowledge’ [15,p.64], pacing of content [16], and starting points for curriculum [14]. This type of learning requires a social transformation as measured by whose voice can be heard in the classroom and ‘opportunities to use math as an analytics tool to critique society’ [2,p.6].

Gutiérrez [2] situated these four dimensions more broadly, namely, ‘in society’ or in a ‘community’ [p.6]. In discussion of power, Gutiérrez [10] positioned the distribution of power in the contexts of the classroom, future schooling, everyday life, and the global society. In this paper, we focus on the classroom as a stepping-stone to discuss alignment of inquiry pedagogies to these dimensions of equity. We utilise these four dimensions as a framework to discuss how active learning pedagogies, and inquiry learning specifically, could have the potential to increase access, lead to higher achievement, provide opportunities for students to reflect on their identities, and attune students to power dynamics in their mathematical community: the classroom. We acknowledge that just using inquiry learning alone may not fully address equity, especially if there is not a change to the system outside the classroom or if students do not have

opportunities to question power distribution and dynamics in the greater society. The purpose of our theoretical exploration is to investigate inquiry learning as an entry point towards a more equitable classroom, ultimately to move towards a more equitable society.

Overview of Inquiry Learning

While this paper reports on teaching through inquiry, we see this pedagogy as a subset of a collection of pedagogies termed *active learning*. Pedagogical techniques used in active learning vary between instructors, including group work, think-pair-share, student presentations, project-based learning, and many other teaching techniques. Freeman et al., [17] reported that active learning techniques have a strong positive impact on student learning. Similarly, Kogan and Laursen's [18] study indicates that 'the benefits of active learning experiences may be lasting and significant for some student groups, with no harm done to others' [p.197].

Under the umbrella of active learning pedagogies, there have been numerous studies on the effects of inquiry-based learning or inquiry-oriented instruction. Even though there is not a consistent definition of inquiry teaching, there are teacher and student practices in the classroom that are essential to inquiry. For example, the Academy of Inquiry-Based Learning describes the philosophy of this pedagogy through student actions: 'students (a) are deeply engaged in rich mathematical tasks, and (b) have ample opportunities to collaborate with peers (where collaboration is defined broadly)' [19].

The IOI description by Rasmussen & Kwon [7] encompasses teacher activity and student activity. With respect to teacher activity, 'teachers routinely *inquire* into their students' mathematical thinking and reasoning' [7,p.2], which has three functions:

First, it enables teachers to construct models for how their students interpret and generate mathematical ideas. Second, it provides opportunities for teachers to learn something new about

particular mathematical ideas, in light of student thinking. Third, it better positions teachers to build on students' thinking by posing new questions and tasks. [7,p.2]

With respect to student activity, 'students learn new mathematics through *inquiry* by engaging in mathematical discussions, posing and following up on conjectures, explaining and justifying their thinking, and solving novel problems' [7,p.2]. This has two functions: 'to enable students to learn new mathematics through engagement in genuine argumentation' and 'to empower learners to see themselves as capable of reinventing mathematics and to see mathematics itself as a human activity' [7,p.2].

Although teacher actions and student actions are distinguished from each other above, we claim that it is not possible to describe students' potential actions independently from the instructor's role in designing and leading an inquiry-based course. This unifying feature of inquiry led Cook et al. [1] to identify six themes of such courses, which we discuss next, contextualizing each theme with excerpts from other researchers' work.

Six Themes of Inquiry

The first theme is *Student-Instructor Relationship* where the instructor asks about student thinking [20] and students can express their own ideas while the instructor listens [8]. Kuster et al. [20] argued that 'questions that require students to engage in problem solving activity affords the instructor opportunities to inquire into student thinking and reasoning' [p.8]. The second theme is *Doing Mathematics* where students participate in authentic mathematical experiences. Cook et al. [1] also describe a third theme called *Student Ownership* where learners are responsible for creating, generating, and developing their own knowledge, either by themselves or with instructors' encouragement. This knowledge is built from their prior knowledge, which they labelled as *Knowledge Building*. Kuster et al. [20] also see this as a fundamental part of IOI and they refer to it as 'building on student contributions' [p.6]. As part of knowledge creation,

students are given opportunities to provide explanations and justifications of their thinking while others listen to and attempt to understand the ideas being discussed or presented, termed *Peer Involvement* by Cook et al. [1]. In Laursen et al. [8], students in IBL courses reported often participating in activities such as asking questions, evaluating other students' work, and working together in class. Kuster et al. [20] also identified students 'being engaged in one another's thinking' as a characteristic of IOI.

According to Cook et al.'s [1] exploration of existing studies, an outcome of their aforementioned features of inquiry is that IBL or IOI is better aligned to how people learn. Similarly, Laursen et al. [8] reported higher 'cognitive gains in understanding and thinking, affective gains in confidence, persistence, and positive attitudes about mathematics, collaborative gains in working with others, seeking help and appreciating different perspectives' [p.409] in students from IBL courses compared to those in non-IBL sections of the same courses. Notably, Laursen et al. [8] also found that in IBL courses, both men and women's attitudes about mathematics improved as well as their interest in pursuing mathematics, but the women had greater gains in these areas than men. Cook et al. [1] categorized this sixth theme as *Student Success*.

The above themes are not meant to be taken as an exhaustive list of features of IBL/IOI teaching practices; they are still under development and undergoing revisions. However, the broadness of Cook et al.'s [1] six themes has motivated us to use them in our preliminary theoretical framework that aims to explore the alignments between IBL/IOI features and the Four Dimensions of Equity by Gutiérrez [2].

Alignment of IBL/IOI with the Four Dimensions of Equity

With this proposed framework, we put forth the claim that, as a pedagogical practice, inquiry learning can be used to promote equity by providing students access and chances to explore their identities, with the hopes of both a shift in both power and achievement in the course. Our exploration originated from several reports, particularly Laursen et al.'s [8] assertion that 'IBL benefits all students even as it levels the playing field for women' [p.415]. Their study documented ways in which IBL can increase achievement in and positive attitudes of mathematics among students. To explicate how the described features of IBL/IOI might provide a more equitable experience for students studying mathematics, we describe some features of IBL/IOI and situate them relative to the Four Dimensions of Equity.

Table 1 shows a summary of the alignment. The first part of the sentence is housed under one of the six themes of inquiry; the sentence continues in the cell that represents the intersection of the inquiry theme and the equity dimension. For example, we theorise that Student Ownership and Power are aligned because: 'When all students are invited to participate in the mathematical classroom community... the power dynamic shifts from instructor as the only source of knowledge to students as producers and users of knowledge.' We further explain parts of the table using some examples.

<< TABLE 1 SHOULD APPEAR NEAR HERE >>

Access

Gutiérrez's [2] definition of equity included a 'classroom environment that invites participation' [p.5] as a tangible resource to access. IBL/IOI pedagogies revolve around a classroom environment that invites and encourages all students' participation in doing, discussing, and presenting mathematics (*Peer Involvement*). When all students are given opportunities to be

active participants in the mathematical community of the classroom (*Doing Math*), students are given an additional access point to learn because they are given the chance to provide explanations and justifications of their thinking processes. Others then listen and attempt to understand the ideas being discussed or presented, which can allow them to build their own mathematical knowledge (*Knowledge Building*). We believe that these opportunities give all students the chance to be exposed to other ways of thinking which can result in richer learning experience for them.

Nasir et al. [21] provided characteristics of classroom practices that support equity: ‘Powerful classroom practices include those that foster student-centred discourse, student exploration of mathematical ideas, and on-going feedback’ [p.17]. Inherent in the on-going feedback is the *Student-Teacher Relationship*: the instructor’s responsibility of inquiring into student thinking and ‘fostering and facilitating productive student discourse’ [21,p.17].

Achievement

Gutiérrez [2] referred to Achievement as a measure of ‘how well students can play the game called mathematics’ [p.6]. In other words, this dimension relates not only to student performance on exams and standardized tests, but also considers a student’s mathematical ‘story.’ This can refer to measures such as whether students continue taking mathematics courses or whether they choose a mathematical career.

When all students are encouraged to create, generate, and develop their own knowledge (*Student Ownership*), confidence in doing mathematics and participation in class may be positively affected. Laursen et al. [8] demonstrated that students in IBL courses increased in student performance as well as other measures related to this definition of achievement.

Additionally, they found that learning gains were found in IBL sections over non-IBL sections of

the same course; not only improvements in course performance, but gains in confidence, persistence, and enjoyment of mathematics (*Student Success*) [8]. Some of these outcomes may lead to Gutiérrez's [2] measures of Achievement, namely 'course taking patterns, majoring in math, and having a math-based career' [p.5]. Kogan and Laursen [18] also reported that students in IBL courses were positively impacted to enroll in more mathematics courses, which aligns with this dimension of equity.

Identity

We claim that the *Peer Involvement* theme of IBL/IOI aligns with Gutiérrez's [2] definition of Identity. When students are actively engaged with each other and each other's thinking (*Peer-Involvement*), it can lead to a shift in mathematical identity. Hassi's [22] qualitative study of students reflecting on their IBL learning experiences supports our claim. In that study, students talked about 'the role of the social environment in an IBL class for gaining or verifying their self-esteem or self-confidence' [p.60]. In addition, Oppland-Cordell & Martin [13] write that

The ways in which individuals continuously construct identities of participation and non-participation over time in [communities of practice] is related to how they position themselves, how others position them, and how such positionings are related to their histories and experiences in the broader contexts in which [communities of practice] are embedded. [p.24]

At the secondary level, Boaler and Greeno [23] contrasted students who learned by working through rote problems in a textbook with students who learned through mathematical discussions (*Peer Involvement*). They found that in discussion-based classes, students were required to contribute more aspects of their selves (as compared to non-discussion-based), which can be done through reflecting on community participation and family relationships. Hassi and Laursen [9] claimed that when students present and evaluate each other's work, students have

heightened perceptions of themselves as mathematical learners, and thus can develop their mathematical identities. This is further evidence for the connection between *Peer Involvement* and Identity.

Power

Gutiérrez [2] thinks of student voice as a fundamental part of the power dimension; inquiry is changing whose voice is primarily present in the classroom. Instructors are responsible for facilitating student discussion and presentation of the problems [24,25]. When given opportunities to provide explanations and justifications of their thinking while others listen to and attempt to understand the ideas being discussed or presented (*Peer Involvement*), power shifts to the students because they decide on ‘what counts as acceptable knowledge’ [15,p.66]. Additionally, the power dynamic in the classroom shifts because student learning is dictated by what they already know as opposed to what the instructors assume they know (*Knowledge-Building*).

The instructor is the primary architect of the problems worked on [16], and when the tasks assigned include problem-posing, students create and solve their own problems (*Doing Math*). In this way, the instructor is orchestrating students’ investigations into their own problems. In this scenario, students have power in deciding parts of the curriculum.

The instructor’s main role is not as a problem-solver, but as an expert participant that guides students to generate, create, and develop their own knowledge (*Student Ownership*). As such, ‘the pace of the course [is] set by students’ movement through this sequence rather than pegged to a pre-set schedule’ [16,p.iii]. In doing this, the instructor signals that the students’ thoughts, beliefs and contributions are a valued part of the learning process and removes themselves as the sole source of knowledge in the classroom. If we agree that *Doing Math*, *Peer*

Involvement, Student Ownership, and Knowledge-Building are components of inquiry teaching, then this represents a substantial shift of the power dynamic from instructor to students.

Future Steps

The theoretical framework we put forth in aligning inquiry pedagogies to equity teaching is our attempt to understand some explicit ways of achieving equity in classrooms. We aim to corroborate the proposed alignments through empirical studies, by learning from students' and teachers' experiences in classrooms that implement inquiry pedagogies, as well as non-inquiry pedagogies. With the understanding that achieving equity in the mathematics classroom is a multi-dimensional problem that requires a multi-dimensional solution approach, we would like to look at other non-inquiry factors that could affect equity. For example, in the class presented in the Motivation section, students engaged in activities that do not fall into the inquiry descriptions above. In this class, students were encouraged to explore mathematical creativity using the *Creativity-in-Progress Rubric on Proving* [26,27]. Students were also required to write weekly reflections on topics such as importance of discussions, effects of inquiry-based learning on student achievement, mathematical creativity, and their perceptions of their performance in the course. The inquiry descriptions above require opportunities to collaborate, but not they do not specify how these collaborations are determined. In this course, the instructor grouped the students based on her perceived notions of their empathy, friendships, and whether they were more introverted or extroverted, rather than randomly or by ability. We also acknowledge that inequities may arise in the *Peer Involvement* component of inquiry [28] as students are interacting with each other. Lastly, we surmise that instructor beliefs could impact equity. Thus, further research needs to be done on possible inequities of inquiry learning.

As a starting point, the theoretical framework we put forth could help describe equitable experiences for all our students regardless of race, gender, ethnicity, social class, sexual orientation, or language. We believe, '[e]quitable classrooms are reflections of a pedagogical, political, and moral vision' [29,p.526]. Hence, to deepen equity in the field of mathematics, we aim to explore more implementation of inquiry pedagogical techniques that integrate content allowing students to use mathematics to critically analyse social justice issues. We believe this particular content consideration with the intent to extend our theoretical frameworks will help achieve equity beyond the classroom and towards the global society.

For instructors who are not ready or cannot (fully) change the curriculum of their class, we claim that by merely engaging in practices of IBL/IOI, we can start to move towards teaching for equity and thinking about students in a more equitable way. That is, engaging in practices of IBL/IOI is an entry point towards engaging in equitable practices.

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	Access	Achievement	Identity	Power
Student-Teacher Relationship When instructors are enabled to have a deeper understanding of student thinking...	...students are given an access point to learn because this helps instructors identify and address student concerns.	...students' learning, confidence, enjoyment of mathematics, and participation in class may be positively affected.	...they may see students as mathematical learners, which may impact how students see themselves as mathematical learners.	...the power dynamic in the classroom shifts since the instructor is concerned with student thinking and not just covering material.
Doing Math When all students are invited to participate in the mathematical classroom community...	...there is an access point to learn since they are given the chance to do, discuss, and present mathematics.	...students may retain more content by participating and building on others' contributions.	...students can reflect on their own mathematical identities as a member of the community.	...the power shifts from instructor as the only source of knowledge to students as producers and users of knowledge.
Student Ownership When all students are encouraged to create, generate, and develop their own knowledge...	...there is an access point to learn because they can work in a way that is different from a prescribed manner.	...there may be gains in learning, confidence, mathematics enjoyment, and class participation.	...students can reflect on their experiences to deepen how they see themselves as mathematical learners.	...the power shifts because students shape traditionally instructor-led components (pacing and content delivery).
Knowledge-Building When all students are encouraged to use prior knowledge to build new knowledge...	...instructors honor what students already know, encouraging an asset perspective instead of a deficit perspective.	...they add to their own understanding, which may lead to gains in learning, confidence, mathematics enjoyment, and class participation.	...students can reflect on their mathematical experiences because they can see the progression in their construction of knowledge.	...the power shifts since the classroom is guided by what they already know as opposed to what instructors assume they know.
Peer Involvement When all students provide justifications while others listen and attempt to understand...	...students are given an access point to learn because they are exposed to other ways of thinking.	...students may achieve together and carry that style of group learning to subsequent courses.	...students' perceptions of their abilities are heightened as they observe how others react to their ideas.	...the power dynamic shifts as students lead the class and ask each other questions, as well as asking the instructor.
Student Success Since IBL/IOI can lead to increased student success...	... and broader access to learning for women, men, low-achieving and first-year students.	... students' career choice and course-taking patterns may be affected.	... students may identify themselves as more of a mathematician or enjoy mathematics more.	... distribution of power in the global society may shift due to a more diversified STEM force.

Table 1: Alignment of Equity and Inquiry